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You go First! – Coordination Problems and the Standard of Proof in Inquisitorial Prosecution

Abstract:

The prosecution of criminals is costly, and subject to errors. In contrast to adversarial court procedures, the prosecutor is regarded as an impartial investigator and aide to the judge in inquisitorial justice systems. We show in a sequential prosecution game of a Bayesian court that a strategic interaction between these two benevolent enforcement agents exists where each player hopes to freeride on the other one´s investigative effort. This gives rise to inefficient equilibria with excessive operating and error costs. Moreover, we will demonstrate that our results are sensitive to the applied standard of proof and that, more disturbingly, the inefficient outcome becomes more probable when the conviction threshold is raised. Applying the concept of 'beyond reasonable doubt', we analyze the impact of the standard of proof and other legal policy instruments on type I and type II errors and operating costs.

JEL-Classification: K14, K41

Keywords: criminal justice, reasonable doubt, litigation, court errors

1. INTRODUCTION

"Hannemann, geh du voran!" (engl. 'Hannemann, you go first') – a German saying[1]

The prosecution of criminals is costly, and potentially erroneous. Given that one´s innocence or guilt to a crime is basically private information, authorities may mistakenly convict an innocent person or set free the true offenders. It is well-understood that such adjudicative errors produce a cost to society (see, e.g., HARRIS 1970, TULLOCK 1994, RIZZOLLI 2019), and that legal institutions should be designed in a way to minimize error and operating costs (see Spier, 2007, pp. 282 for an overview).

In adversarial (or 'partisan') legal systems, the prosecutor and the defendant´s advocate are taking opposing sides as they try to reveal no information that weakens the own prospect of winning while being tough with the other party. In the words of TIROLE and DEWATRIPONT (1999, p. 3), however, "this conflict generates useful information" in front of the court and thereby enhances the decision-making of judge or jury. In inquisitorial legal systems, the prosecutor is not regarded as an advocate of one specific party to the case, but is expected to support the court in its search for the substantive truth (see, e.g., GAROUPA 2011). In this perspective, the prosecutor is perceived as an impartial aide who runs the investigations (so-called '*Herrin des Ermittlungsverfahrens*') under the supervision of the benevolent judge (see SPIER, 2007, pp. 313).

[1] This German saying "Hannemann, you go first" describes a situation where a group of people share a common goal, but everybody prefers that one of the others makes the unpleasant first move.

In addition to the adversarial or inquisitorial tradition of prosecution, many jurisdictions have established the conviction threshold of 'beyond reasonable doubt' which requires that a defendant can only be convicted as guilty when no reasonable doubts about the case remain.[2] Evidently, this standard of proof in criminal procedure is more restrictive than the 'more probable than not'-standard often applied for civil law cases (see, e.g., KAYE 2002). This reflects the widely accepted view that wrongful convictions of innocent individuals are regarded as more harmful to society than wrongful acquittals, and thus a higher conviction threshold is required to mitigate error costs (for an overview, see ANDREONI 1991, and TSUR 2017).

In this paper, we will show that the strategic interaction between two benevolent enforcement agents, the judge and the prosecutor, still gives rise to inefficient equilibria in inquisitorial criminal procedure. Moreover, we will demonstrate that the inefficient outcome becomes more probable when the conviction threshold is raised. In the following, we develop a sequential prosecution game in which both the prosecutor and the judge could perfectly reveal the true guilt of the defendant, but investigations create private effort costs. As a consequence, a freeriding dilemma unfolds where each enforcement agent hopes to benefit from the other´s investigative effort. Following the formal concept of 'beyond

[2] For example, this high standard of proofs is expressed in Coffin v. US, 156 U.S. 432 (1895) and Woolmington v DPP (1935) UKHL 1. German criminal justice order requires for a conviction the 'firm belief of the judge' (§ 261 StPO), which is often regarded as being specified by the German High Court ruling to "a usable degree of certainty which puts silence to any remaining doubts without fully eliminating them" (BGH 1993, IX ZR 238/91).

reasonable doubt' by TSUR (2017), we analyze the impact of the standard of proof and other legal policy instruments on type I and type II errors and operating costs.

This paper is organized as follows: chapter 2 provides the basic framework of the model. We then conduct a normative analysis in chapter 3, and identify the equilibria to the game in chapter 4. In chapter 5, we discuss different policy instrument. Chapter 6 concludes.

2. INQUISITORIAL PROSECUTION MODEL

Imagine a person, the defendant, who is accused of having committed a crime to society. Law demands the guilty defendants to be put to jail while the innocent defendants are to be acquitted. Given this setting, consider the criminal justice system as a prosecution game with two players, the prosecutor P and the Judge J, who seek to determine the actual guilt of the defendant. The defendant can either be guilty *(G)* or innocent *(I)*, and the ex-ante probability of a guilty defendant is defined as γ. All this is common knowledge.

Both players (P and J) may investigate the evidence to the case during the criminal proceedings. For simplicity, we assume that the investigative effort perfectly reveals the defendant´s guilt or innocence, but induces effort costs c_P for the prosecutor and c_J for the judge. Due to the superior resources of the prosecutorial office and its closer cooperation with the police force, the prosecutor can investigate the case at lower costs compared to the judge. Thus, we specify $c_P < c_J$. The prosecutor can then either drop the case or charge the defendant and move to court. In courtroom, the judge may either investigate the case himself, which then leads to a correct decision, or decide about the conviction or acquittal of the defendant without (further) investigations. However, whether the case was actually investigated beforehand or simply passed on to the judge remains private information of the charging prosecutor.

The law, and society, demand the avoidance of wrongful convictions (type I error) and wrongful acquittals (type II errors). We assume that both players adhere to these goals of society. Thus, the judge and the prosecutor receive a disutility of H if a truly guilty defendant is wrongfully set free, and a disutility of αH, with $\alpha > 1$, if an innocent defendant is actually put to jail. We thus follow the general notion that most societies consider wrongful convictions to be more harmful than wrongful acquittals. With regard to the social costs of errors, the exogenous parameter α can be interpreted as the number of wrongful acquittals of guilty defendants that can be accepted in order to avoid the erroneous conviction of a single innocent person (see TSUR 2017, p. 198). From the perspective of society, this parameter thus determines the decision standard of 'beyond reasonable doubt' in criminal procedure: given the above specified error costs and the exogenous probability of guilt, a court will convict the defendant if $-(1-\gamma)\alpha H \geq -\gamma H$, which gives the 'beyond reasonable doubt'-threshold as $\gamma \geq \frac{\alpha}{1+\alpha} = \gamma$.

In addition to these goals of society, the prosecutor further receives a disutility of L if she loses 'her case' in court. This captures reputational concerns of the prosecutor with regard to her peers, future defendants, and her superiors. All players also incur trial costs T when the case is brought to court. Overall, judges and prosecutors are assumed to be risk-neutral, and they maximize their expected utility function.

The noncooperative prosecution game consists of three stages as shown in *Figure 1*: The prosecutor´s decision to investigate (stage 1), her decision to bring the case to court (stage 2), and the decision by the judge (stage 3).

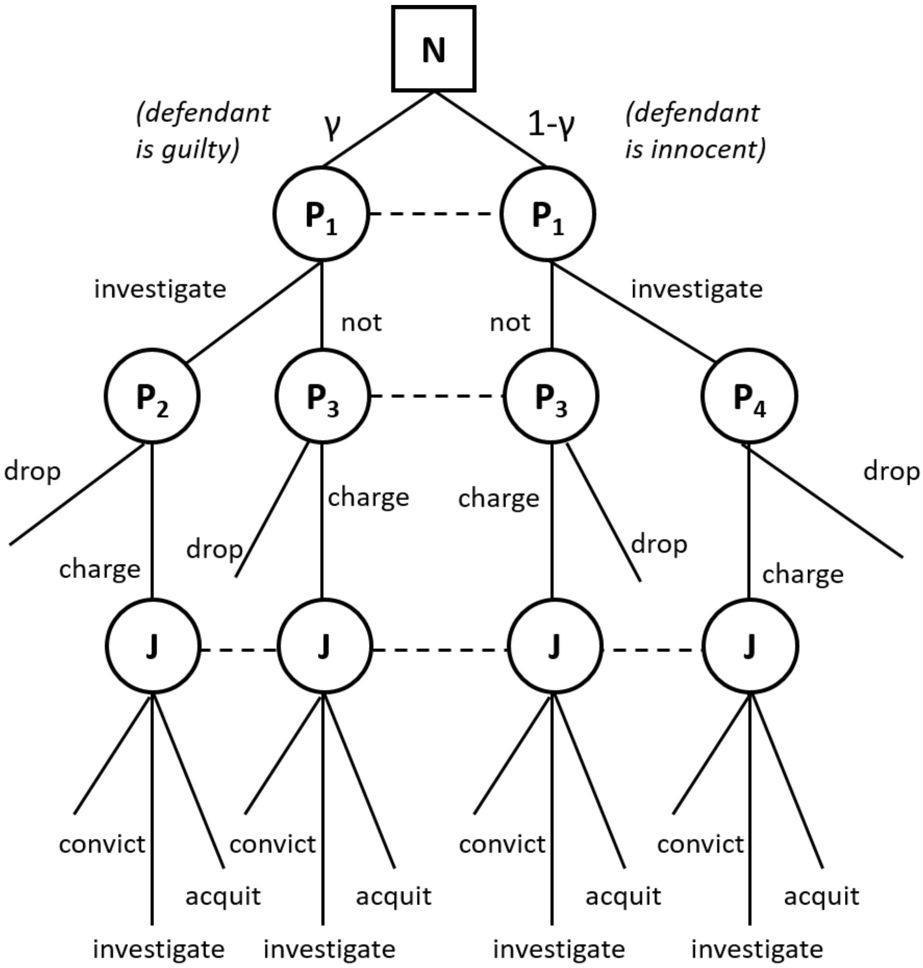

Figure 1. Inquisitorial prosecution game.

At the beginning, nature (N) determines the defendant´s true type, be it guilty or innocent. At stage 1, the prosecutor may then decide to investigate and learn the true type of the defendant with certainty, or she proceeds without any further generation of evidence. She may at stage 2 either drop the case, which ends the game, or proceed and move to court. At stage 3, the judge does not know whether the prosecutor actually put effort into the investigations of the case. The judge may thus either run investigations himself, which perfectly reveals the defendant´s type and leads to a correct decision with certainty, or decide about the defendant´s guilt based on his ex-post beliefs.

3. NORMATIVE ANALYSIS

From the perspective of society, the criminal justice system is to minimize error and operating costs. Error costs refer to the social loss due to wrongful convictions and erroneous acquittals, including the negative impact on deterrence. Operating costs refer to all resources consumed for trials and investigative effort in criminal procedures. Overall, costly trials should only be held in order to convict the truly guilty defendants. Furthermore, any necessary investigations into the case should be run before trial by the prosecutor due to her cost advantage and her ability to select cases for court. Dependent on the above specified *reasonable doubt*-bound (γ), we can thus determine when it is socially optimal to bring the case to court and when investigations are desirable.

Assume that the ex-ante probability of a guilty defendant is rather low and the *beyond reasonable*-doubt-bound is not met, thus $\gamma < \gamma$ applies. Then, dropping the case yields costs γH while prosecutorial investigations produce costs $c_P + \gamma \cdot 2T$, due to fact-finding costs c_P and trial costs T for both judge and prosecutor.[3] Thus, efficiency requires such a case to be investigated by the prosecutor if $\gamma > \frac{c_P}{H - 2T} = \underline{\gamma}$ holds.

[3] Note that a blind charge can never be desirable as the judge would acquit the defendant when the evidence is below the conviction threshold.

When the reasonable doubt-bond is now met, $\gamma > \gamma$, the court is expected to convict the defendant. Charging the defendant then produces error and operating costs $(1-\gamma)\alpha H + 2T$ due to some wrongful convictions and the costs of trial. Alternatively, the prosecutor rules out court errors by investigating the case which causes costs of $c_P + \gamma 2T$. Consequently, investigations of such cases are socially preferable if $\gamma < 1 - \frac{c_P}{2T + \alpha H} = \bar{\gamma}$ applies.

We conclude that for the first-best solution, cases with a rather low probability of guilt (i.e., $0 < \gamma < \underline{\gamma}$ and $\gamma < \gamma$) are to be dropped while cases with overwhelming evidence (i.e., $1 > \gamma > \bar{\gamma}$ and $\gamma > \gamma$) lead to a conviction without further fact-finding effort. All remaining cases require investigations by the prosecutorial office.

4. POSITIVE ANALYSIS

In this section, we will derive Nash equilibria of the game depicted in *Figure 1* and examine whether these equilibria are as well Perfect Bayesian equilibria. To find all Nash equilibria, we need to set up the strategic form of the game. The strategic form of a game consists of the set of players, the set of strategy combinations, and the set of combinations of payoffs which the players attach to each strategy combination.

Player J has three feasible actions at each decision node in his single information set: J can investigate the case, thereby reveal the actual type of the defendant, and resolve the case according to the collected information. Due to the assumptions made with regard to J's incentives, the ultimate decision under full information is trivial. Moreover, J could convict or acquit the defendant without first examining the case. Hence, J's strategy set is {convict, acquit, investigate}.

Player P has four information sets. At P_1, she decides whether to investigate or not. To investigate the case would reveal the true type of the defendant, in which case P could either drop the case (and end the game) or bring charges, handing over the case to J, see P_2 and P_4, respectively. If she fails to investigate, she has to make the same decision, but without knowledge, at her decision nodes in information set P_3. Hence, P's strategy set is the Cartesian product {investigate; not}x{charge; drop}. Obviously, P has 16 strategies. Given the specified order of the information sets, one example would be (not, drop, charge, drop). This strategy represents the plan to not investigate the case and bring charges (at P_3), while at P_2 and P_4 player P would have dropped the case.

The strategic form could, therefore, be represented by a 16x3 bi-matrix indicating the players' payoffs. However, many cells in this bi-matrix would contain information which is redundant. After some eliminations of redundant information and dominated strategies (see Annex A1), the reduced strategic form in *Figure 2* is an adequate representation of the game. The Nash equilibria identified in this reduced strategic form describe the Nash equilibria of the extensive form game. For brevity, we use the following notation for the described actions investigate (*inv*), not investigate (*n*), charge (*ch*), drop (*dr*), convict (*co*) and acquit (*ac*).

		Judge		
		investigate ***inv***	not, convict ***n, co***	not, acquit ***n, ac***
Prosecutor	Investigate ***inv***	$-\gamma T-\gamma c_J$ $-\gamma T-c_P$	$-\gamma T$ $-\gamma T-c_P$	$-\gamma(H+T)$ $-\gamma(H+T)-c_P$
	not, charge ***n, ch***	$-T-c_J$ $-T-(1-\gamma)L$	$-T-(1-\gamma)\alpha H$ $-T-(1-\gamma)\alpha H$	$-T-\gamma H$ $-T-\gamma H-L$
	not, drop ***n, dr***	$-\gamma H$ $-\gamma H$	$-\gamma H$ $-\gamma H$	$-\gamma H$ $-\gamma H$

Figure 2. Reduced payoff matrix.

A Perfect Bayesian Equilibrium (PBE) in this game of asymmetric information consists of the strategies $\{s_P; s_J\}$, with $s_P \in [(inv);(n,ch);(n,dr)]$ and $s_J \in [(inv);(n,co);(n,ac)]$, and the judicial beliefs μ about the defendant´s guilt given the case is brought to court such that, at any stage of the game, strategies are optimal given the beliefs, and the beliefs are obtained from the equilibrium strategies using Bayes´ rule (FUDENBERG/TIROLE 1999).

In this game, six candidates for a PBE in pure strategies exist contingent on the ex-ante probability of a guilty defendant. In the following, we present the results from low to high values of the a priori probability of a guilty defendant, γ.

For low values of the prior γ, one PBE in pure strategies exists in which the prosecutor always drops the case and no further investigations into facts are made. Such cases are below the *reasonable doubt*-threshold for a conviction, $\gamma < \gamma$, and guilt of the defendant is so unlikely, $\gamma < \frac{c_J}{H}$, that investigations are not favorable to the judge.

Proposition 1. *The strategies* $\{(n,dr);(n,ac)\}$ *are (i) a Nash Equilibrium which constitutes (ii) a PBE if also* $\gamma < \gamma$ *,* $\gamma < \frac{c_j}{H}$*,* $\gamma < \frac{c_P}{H-T}$ *and beliefs* $\mu = [0,1]$ *apply (PBE No.1).*

For intermediate values of the prior γ , the game shows up to three candidates for PBE in pure strategies. As two of these equilibria can coexist, also one PBE in mixed strategies can then be determined.

The first candidate (PBE No. 2) shows the same outcome as PBE No.1, that is, no investigations occur and the prosecutor never charges. This equilibrium applies if for some intermediate probabilities of the defendant´s guilt the judge would rationally choose investigations while litigation is too costly for the prosecutor. For specific combinations of cost parameters (c_P, c_J, H, T), the condition $\frac{c_P}{H-T} > \gamma > \frac{c_J}{H}$ holds. Given that we generally assumed a cost advantage in investigations for the prosecutor, this specific outcome occurs only if litigation costs T are relatively high compared to the error costs of wrongful acquittals, H. As litigation costs are treated as sunk costs by the judge but not by the prosecutor, the judge may decide to investigate the case while the prosecutor is better off dropping the charge. Then, PBE No. 2 actually occurs.[4]

[4] Note that PBE No. 2 may not coexist with PBE No.3 and PBE No.4, and thus is irrelevant for any mixed strategy equilibrium.

Proposition 2. *The strategies* $\{(n,dr);(inv)\}$ *form (i) a Nash Equilibrium if* $\gamma < \frac{c_p}{H-T} < \underline{\gamma}$ *and* $\gamma < \frac{T+L}{H+L}$ *hold. (ii) This equilibrium is a PBE if also* $\gamma > \frac{c_j}{H}$, $\gamma < 1 - \frac{c_j}{\alpha H}$ *and beliefs* $\mu = [0,1]$ *apply (PBE No.2).*

For (most)[5] intermediate values of the prior γ, two PBE coexist in pure strategies. In either equilibrium, one player investigates and the other player free-rides. From an efficiency perspective, only the equilibrium (PBE 4) where the prosecutor looks into the case is efficient, as she bears the lowest fact-finding costs and the same court ruling is obtained.

Proposition 3. *The strategies* $\{(n,ch);(inv)\}$ *form (i) a Nash Equilibrium if* $\gamma > \frac{T+L}{H+L}$, $\gamma > \frac{L+T-c_P}{L+T}$, $\gamma > \frac{c_J}{H}$ *and* $\gamma < \frac{\alpha H - c_J}{\alpha H}$ *hold. (ii) This equilibrium is a PBE given the belief* $\mu(G \mid ch) = \gamma$ (PBE No.3).

Proposition 4. *The strategies* $\{(inv);(n,co)\}$ *form (i) a Nash Equilibrium if* $\gamma < 1 - \frac{c_P}{T+\alpha H} < \bar{\gamma}$ *and* $\gamma > \frac{c_P}{H-T} < \underline{\gamma}$ *hold. (ii) This equilibrium is a PBE given the belief* $\mu(G \mid ch) = 1$ (PBE No.4).

When those two equilibria in pure strategies coexist, there is also a PBE in mixed strategies where the prosecutor and the judge randomize their investigation effort. In this case, the prosecutor always drops the case when she learns the defendant to be innocent,

[5] For both players, the decision to investigate is contingent on the prior, but each player shows a slightly different range of γ in which investigations are desirable. Two PBE (and the mixed strategy solution) exist only in this overlap of the established ranges.

given she looked into the case, and she charges the defendant otherwise. The judge either investigates the case himself and comes to the correct decision, or convicts blindly.

Proposition 5. *Given the two Nash Equilibria in pure strategies* $\{(n,ch);(inv)\}$ *and* $\{(inv);(n,co)\}$ *coexist, then there exists a PBE in mixed strategies with P´s probability of effort* $\phi^*_P = \frac{(1-\gamma)\alpha H - c_J}{(1-\gamma)(\alpha H - c_J)}$ and J´s probability of effort $\phi^*_J = \frac{(1-\gamma)(\alpha H + T) - c_P}{(1-\gamma)\alpha H - (1-\gamma)L}$, and the judicial belief $\mu = \frac{\gamma}{\gamma + (1-\phi_P)(1-\gamma)}$ (PBE No.5).

Assume the above established requirements for the two Nash equilibria in pure strategies hold. Then prosecutor can choose her investigative effort in a way to turn the judge indifferent between own investigations or blind convictions.[6] The judge is indifferent when $-c_J - T = -(1-\gamma)\alpha H - T$ holds, which yields the threshold $\gamma = 1 - \frac{c_J}{\alpha H}$. Through the randomization of her investigative effort, the prosecutor will induce the ex-post probability of guilt to equal this threshold. We designate the probability that P investigates as $\phi_P \in [0,1]$. As the prosecutor will always charge when she learns the defendant to be guilty and never when she finds the defendant to be innocent, the judge´s posteriori beliefs about the probability of a guilty defendant $\mu = \frac{\gamma}{\gamma + (1-\phi_P)(1-\gamma)}$ thus have to equal the above

[6] Note that P cannot make J indifferent between investigations and blind acquittals, because P´s investigative efforts leads to dropped cases against innocent defendants, which always increases the probability of a guilty defendant γ in the remaining pool of filed cases.

mentioned threshold, this $\mu = \frac{\gamma}{\gamma + (1-\phi_P)(1-\gamma)} = 1 - \frac{c_J}{\alpha H}$ holds. Solving for ϕ_P, the prosecutor randomizes his effort with $\phi^*_P = \frac{(1-\gamma)\alpha H - c_J}{(1-\gamma)\left(\alpha H - c_J\right)}$.[7] In turn, the judge can make the prosecutor indifferent between investigation and blind charge by randomizing his judicial investigative effort. We specify the probability that the judge looks into the case with $\phi_J \in [0,1]$. The prosecutor is thus indifferent when the condition $-c_P - \gamma T = \phi_J\left(-(1-\gamma)L - T\right) + (1-\phi_J)\left(-(1-\gamma)\alpha H - T\right)$ holds. This yields $\phi^*_J = \frac{(1-\gamma)\left(\alpha H + T\right) - c_P}{(1-\gamma)\alpha H - (1-\gamma)L}$ which lies in the interval (0,1).[8]

For high values of the ex-ante probability of a guilty defendant and when the prior exceeds the *reasonable doubt*-threshold, then the nature of the PBE depends on whether the prosecutor prefers a blind conviction of the defendant to dropping the case. In the former case, both players do not investigate the case and all defendants are charged and convicted. Given the little gain through investigations when guilt is highly likely and beyond the *reasonable doubt*-threshold, this outcome is also considered efficient.

Proposition 6. *The strategies* $\{(n,ch),(n,co)\}$ *form (i) a Nash Equilibrium if* $\gamma > 1 - \frac{c_P}{\alpha H + T} < \bar{\gamma}$, $\gamma > \frac{\alpha H + T}{(1+\alpha)H}$, $\gamma > \frac{\alpha H - c_J}{\alpha H}$ *and* $\gamma > \hat{\gamma}$. *(ii) This solution is a PBE for the belief* $\mu\left(G \mid ch\right) = \gamma$ (PBE No.6).

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[7] The mixed strategy for P is zero if the prior already meets the threshold at which J is indifferent.

[8] The numerator is positive for $\gamma < 1 - c_p/(\alpha H + T)$ and always smaller than the denominator for $\gamma < 1 - c_p/(T + L)$, which both holds when the two Nash Equilibria in pure strategies exist.

However, the prosecutor may decide to drop the charge although the defendant´s guilt is beyond *reasonable doubt*, which occurs for $\gamma < \gamma < \frac{\alpha H + T}{(1+\alpha)H}$.

Proposition 7. *The strategies* $\{(n,dr);(n,co)\}$ *form (i) a Nash Equilibrium if* $\gamma < \frac{\alpha H + T}{(1+\alpha)H}$ *and* $\gamma < \frac{c_p}{H-T} < \underline{\gamma}$ *hold. (ii) This equilibrium is a PBE if also* $\gamma > \gamma$, $\gamma > 1 - \frac{c_j}{\alpha H}$ *and beliefs* $\mu(G \mid ch) = [0,1]$ *apply (PBE No.7).*

5. FREE-RIDING BEHAVIOR AND POLICY INSTRUMENTS

The determined PBE show distinct properties with regard to court errors and operating costs. From a welfare perspective, two distinct inefficiencies can be identified: first, the decision to investigate by the prosecutor is affected by her private costs of trial, but her decision does not internalize trial costs for the judge. Consequently, she will charge some cases that, from the perspective of society, should be dropped and she will refrain from desirable investigations in some other cases. Second, a free-riding dilemma occurs in the strategic interaction between the prosecutor and the judge, which leads to inflated operating and error costs. *Figure 3* provides an overview of error types and social costs of the previously established equilibria.

For low and high values of the probability of a guilty defendant, the equilibrium path will be mostly efficient: for low γ, it is socially preferable to drop the case (i.e., PBE 1 applies). A conviction without further investigations is favorable whenever the ex-ante probability of guilt γ is high (i.e., PBE 6 holds). It is well established in the literature (See. e.g., SPIER 2007, p. 264-268) that the private and the social incentive to litigate may differ, and thus may induce the above mentioned inefficiencies at the margin $\underline{\gamma}$ and $\bar{\gamma}$. In the following, we shift our focus to inefficiencies caused by the free-riding dilemma between the prosecutor and the judge.

PBE	Error Costs (error type)	Operating Costs	Free-Riding	Efficient
No. 1	γH (type II error)	0	No	for $\gamma < \underline{\gamma}$
No. 2	γH (type II error)	0	No	yes
No. 3	$(1-\gamma)L$ (type I error)	$2T + c_J$	Yes	no
No. 4	0 (none)	$c_P + \gamma 2T$	Yes	for $\gamma > \underline{\gamma}$
No. 5	$(1-\phi_p - \phi_J + \phi_J \phi_p)\cdot(1-\gamma)\alpha H$ $+(1-\phi_P)\phi_J(1-\gamma)L$ (type I/II errors)	$\phi_P c_P$ $+(1-\phi_P(1-\gamma))(2T + \phi_J c_J)$	Yes	no
No. 6	$(1-\gamma)\alpha H$ (type I error)	$2T$	No	$\gamma > \bar{\gamma}$
No. 7	γH (type II error)	0	No	yes

Figure 3. Court errors, operating costs, and free-riding.

This strategic interaction between the prosecutor and the judge can be described by the well-known battle-of-the-sexes dilemma whenever PBE 3 and PBE 4 coexist: even though both parties seek a common goal, they conflict over the distribution of the rent. In the context of this game, both agents seek to convict the guilty and acquit the innocent, but they prefer that the other party bears the investigative costs. As we assumed that the prosecutor shows lower effort costs, PBE 4 would be socially preferable to judicial investigations (PBE 3). PBE 4 is also favorable to the mixed strategy equilibrium (PBE 5) which gives rise to the risk of both types of errors and may also cause redundant investigations by both parties. In order to avoid such inefficiencies in inquisitorial prosecution systems, it is relevant for policy makers to know how the described free-riding dilemma can be dissolved and how PBE 4 can be implemented for intermediate values of γ.

We consider the following variables as potential policy instruments and discuss their impact on the free-riding problem among the prosecutorial agents: the 'beyond reasonable doubt' conviction threshold ($\gamma = \frac{\alpha}{\alpha+1}$), trial costs T, and the reputation loss L for a losing prosecutor.

The 'beyond reasonable doubt' conviction threshold describes the minimal requirement for the judge´s ex-post belief about the defendant´s guilt in order to justify a conviction. This requirement increases in α, which we interpreted as the number of wrongful acquittals that equal the social cost of one wrongful conviction. It follows the standard debate in the legal sciences (see, among others, ANDREONI 1991, WEINSTEIN/DEWSBURY 2006, TSUR 2017) that α will be higher for more serious punishments. Our prosecution game reveals the disturbing finding that α also intensifies the free-riding problem in inquisitorial prosecution: First, an increasing α also increases the range where PBE 3 and PBE 4, and thus the battle-of-sexes interaction, may occur. [9] For both equilibria, the upper boundary with regard to the ex-ante probability of guilt γ is shifted upward if a

[9] PBE 3 faces the upper bound of $\gamma < 1 - \frac{c_J}{\alpha H}$, with $\frac{\partial \gamma}{\partial \alpha} > 0$, and PBE 4 is bound by $\gamma < 1 - \frac{c_P}{T + \alpha H}$, with $\frac{\partial \gamma}{\partial \alpha} > 0$.

higher α (and thus a higher *reasonable doubt* conviction threshold) applies. This is an intuitive result as higher social costs of type I errors require a higher level of certainty about the defendant´s guilt, which enlarges the range where investigations are preferable to a blind conviction. Second, the error costs of the mixed strategy equilibrium PBE 5 increase in α. Higher values of α affect the randomization strategy of the judge and lead c.p. to a lower probability of investigation effort ($\frac{\partial \phi^*_J}{\partial \alpha} < 0$). As the effort of the prosecutor remains constant ($\frac{\partial \phi^*_P}{\partial \alpha} = 0$), this leads to less investigations into the case. As the judge always convicts the charged defendant if he did not investigate the case, this unambiguously increases the occurrence of wrongful convictions. Both effects show that the 'beyond reasonable doubt' conviction threshold directly impacts the free-riding dilemma. More precisely, a higher 'beyond reasonable doubt' bound, which is usually aligned to the more serious crimes, makes the free-riding problem more prominent, gives rise to inefficient equilibria and c.p. increases the occurrence of type I errors due to the strategic interaction. Policy makers need to be aware that this inefficient free-riding behavior follows the 'beyond reasonable doubt' threshold, and cannot be mitigated by higher requirements for a conviction.

Another potential policy instrument is the design of trial costs. In this sequential game, trial costs also affect both players differently, as the prosecutor can evade such costs by dropping the charge while trial costs are always sunk for the judge. In fact, an increase in trial costs narrows the range for the inefficient PBE 3.[10] At the same time, however, the range for the desirable outcome PBE 4, where only the prosecutor investigates the case, is shifted upward.[11] As a consequence, some cases that were previously investigated by the prosecutor will now be dropped as the equilibrium path changes to PBE 1 or PBE 2. In other words, the reduction of the potential free-riding problem comes at the cost of more wrongful acquittals (type II errors).

The third variable of interest is the reputational concern L of the charging prosecutor. A prosecutor, who wants to avoid losing a case that she actively chose to bring to court, is clearly incentivized to make the correct choice in the first place. As investigations into the case are her means to achieve this, strong reputational concerns among prosecutors may induce the desired outcome: an increase in reputation loss L unambiguously narrows the range of the inefficient PBE 3, [12] but does not affect the desired outcome PBE 4. In other words, the prosecutor will investigate more cases for intermediate values of the prior γ, which is efficient. However, as long as PBE 3 and PBE 4 still coexist for some values of the ex-ante probability of guilt γ, the outcome of the mixed strategy equilibrium PBE 5 deteriorates in this range. The higher reputational loss will affect the

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[10] For the lower boundaries $\gamma > \frac{T+L}{H+L}$ and $\gamma > \frac{L+T-c_P}{L+T}$, $\frac{\partial \gamma}{\partial T} > 0$ applies while the upper boundary remains unaffected for changes in T.

[11] Both bounds $\gamma < 1 - \frac{c_P}{T+\alpha H}$ and $\gamma > \frac{c_P}{H-T}$ show $\frac{\partial \gamma}{\partial T} > 0$.

[12] For the lower bounds $\gamma > \frac{T+L}{H+L}$ and $\gamma > \frac{L+T-c_P}{L+T}$, $\frac{\partial \gamma}{\partial L} > 0$ applies while the upper bound is unaffected.

mixed strategy of the judge and c.p. reduce the probability that the judge investigates the case.[13] As a consequence, more wrongful convictions occur. We conclude that only for strong reputational concerns by prosecutors can the free-riding problem be ruled out. Otherwise, there is a trade-off between less free-riding and a higher occurrence of type I errors.

6. CONCLUSIONS

The criminal justice system is to minimize error and operating costs. Most societies regard wrongful convictions (type I errors) as particularly harmful. Institutional safeguards to mitigate such error costs include the higher standard of proof of 'beyond reasonable doubt' and, for inquisitorial legal systems, the competence of judges to actively investigate cases themselves.

In this paper, we show that the higher standard of proof amplifies the coordination problem between prosecutors and judges in inquisitorial legal systems. Inquisitorial criminal procedures are described as a sequential game between a benevolent prosecutor and a judge who seek to convict the guilty and set free the innocent defendants. Both may exert private and unobservable effort to learn the defendant´s true guilt or innocence. We further assume that the prosecutor can investigate a case at lower costs due to superior resources. Efficiency thus requires that the prosecutor investigates a case whenever effort and trial costs are lower than the costs of errors.

[13] The mixed strategy $\phi^*_J = \frac{(1-\gamma)(\alpha H + T) - c_P}{(1-\gamma)\alpha H - (1-\gamma)L}$ shows $\frac{\partial \phi^*_J}{\partial L} < 0$.

The identified equilibria of the game are mostly efficient for very low or very high probabilities of guilt of the defendant. In the former case, the prosecutor drops the charge and in the latter case a conviction is obtained without (further) investigations. The positive analysis also reveals that for intermediate probabilities of the defendant´s guilt, a freeriding dilemma unfolds: both enforcement agents are interested in verifying the case and thus making a correct decision, but each player prefers that the other one bears the effort costs. In addition to the two Perfect Bayesian Equilibria in pure strategies, where one player investigates the case and the other freerides, one equilibrium in mixed strategies exist. For this outcome, each player exerts investigative effort with a positive probability, but less than one. This generates the risk that either a doubtful criminal case is never investigated but concluded by a 'blind' conviction or both agents investigate which implies the duplication of efforts. Of the three potential outcomes, only the equilibrium where the prosecutor investigates the case is efficient.

We discuss three policy measures to support the efficient equilibrium:

First, an increase in the 'beyond reasonable doubt' threshold both increases the range where the freeriding dilemma may occur and reduces the probability of judicial effort in the mixed strategy equilibrium. As the probability of prosecutorial effort stays constant, overall adjudicative accuracy is thus degraded. Given that a higher standard of proof is typically applied when the punishment becomes more serious, then higher punishments go along with more wrongful convictions. Policy makers need to be aware that the described coordination problem is not mitigated by a higher standard of proof.

Second, the costs of trial show an appealing property. Trial costs affect the optimal behavior of the prosecutor, but are ignored as sunk costs by the judge. Thus, an increase in (common) trial costs c.p. restricts both the prosecutorial decision to bring a case to court and her incentive to freeride on the judge´s effort. In other words, the range for the inefficient equilibria is reduced at the cost of more wrongful acquittals.

Third, reputational concerns are considered. If the reputational loss for losing a case in courtroom is serious for the prosecutor, then she is incentivized to investigate doubtful cases herself and only select probable charges for court. However, as long as the coordination problem is not perfectly solved, the number of wrongful convictions may actually increase. If the potential reputational damage is sufficiently high, the prosecutor will not risk to freeride and only the efficient outcome remains. The impact of such reputation concerns, interpreted as a cost when a trial is lost, can to some degree be compared to the incentive effect of the British rule of fee-shifting ('loser-pays-all'-rule).

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8. ANNEX

A1 Elimination of redundancies and dominated strategies

First we discuss the strategies of P which contain "not" to investigate as their first entry:

- If P decides, after having not examined the case at P_1, to drop the case (at P_3), then the expected payoffs of both parties amount to $-\gamma H$ each, regardless of what P plans to do at her other two decision nodes, and also regardless of J's plan. Hence, these four strategies of the type (not, x, drop, y), where x and y are either drop or charge, can be summarized without loss of information to one strategy pattern which we, in the bottom row of figure 2, will label "not, drop (*n, dr*)."

- A similar line of argument covers the four strategies in which P plans to hand over the case to J without having it examined initially, i.e., when P chooses not to investigate at P_1, and charge at P_3. Regardless of what P plans for her other information sets, the expected payoffs of the two players only depend on J's decision, so four rows of the 16x3 matrix show identical entries. Hence, we combine these four strategies of P without loss of information into one, labeled "not, charge, (*n, ch*)" in figure 2 (medium row).

Finally, we discuss the eight strategies of P that stipulate an investigation at P_1. After P has investigated the case, the parties' payoffs do not depend anymore on what P would have planned to choose at her information set P_3 (the one that is reached after not investigating the case). Hence, P has only four relevant types of strategies, depending on the decisions made at P_2 and P_4. The payoffs of those four types of investigation strategies depend on the decision of the judge, and are displayed in the table below. The letter x represents the possible decisions at P_3. To show which investigative strategies are dominated, we added the non-investigative strategy type (n, dr) where P does not investigate and always drops the case.

	J investigates	J convicts	J acquits
(inv, ch, x, ch)	-c_P-T-(1-γ)L	-c_P-T-(1-γ)αH	-c_P-T-L-γH
(inv, ch, x, dr) = "(inv)"	-c_P-γT	-c_P-γT	-c_P-γT-γL-γH
(inv, dr, x, ch)	-c_P-γH-(1-γ)(L+T)	-c_P-γH-(1-γ)(αH+T)	-c_P-γH-(1-γ)(L+T)
(inv, dr, x, dr)	-c_P-γH	-c_P-γH	-c_P-γH
"(n, dr)"	(-γH)	(-γH)	-γH

Figure A1. Elimination of dominated strategies.

Remember that T < H applies. The grey shaded cells identify the best strategy of P for a given decision by J. This clearly shows that the six strategies of the types (inv, ch, x, ch), (inv, dr, x, ch) and (inv, dr, x, dr) are never optimal for P and thus are always dominated by one of the other strategies: either by (inv, ch, x, dr) if the judge investigates himself or charges blindly, or simply by the non-investigative strategies of type (n, dr). For the purpose of finding Nash equilibria in pure strategies, we can eliminate dominated strategies, since they will never be best responses. In figure 2, we thus only keep the strategies (inv, ch, x, dr), which for the sake of brevity we label as "investigate (inv)" from here.

A2 Proofs.

Proof Proposition 1. (i) If the Judge acquits the defendant, the prosecutor´s best response is to save effort and drop the case in the first place. If the prosecutor drops the case, all choices of the judge are best responses, given that the case never reaches court. (ii) For (n, dr) being a best response of the judge in the subgame where P charges the defendant, J must prefer acquittal to conviction which requires $-(1-\gamma)\alpha H - T > -\gamma H - T \Leftrightarrow \gamma < \frac{\alpha}{1+\alpha}$ and J does not investigate the case $-c_J - T < -\gamma H - T \Leftrightarrow \gamma < \frac{c_J}{H}$. Also for this subgame, P has still to prefer not to investigate the case, which requires $-\gamma T - c_P < -\gamma H \Leftrightarrow \gamma < \frac{c_P}{H-T}$ ■

Proof Proposition 2. (i) Given that P drops the case, J is indifferent between his strategies. For (n, dr) being the best response to J investigating, first, $-\gamma H > -\gamma T - c_p$ must hold, which gives $\frac{c_P}{H-T} > \gamma$, and second, $-\gamma H > -T-(1-\gamma)L$, which yields $\frac{T+L}{L+H} > \gamma$. (ii) For (inv) being a best response of the judge in the subgame where P charges the defendant, J must prefer investigation of the case to a blind acquittal, $-c_J - T > -\gamma H - T \Leftrightarrow \gamma > \frac{c_J}{H}$, and also to a blind conviction, $-c_J - T > -(1-\gamma)\alpha H - T \Leftrightarrow 1 - \frac{c_J}{\alpha H} > \gamma$.■

Proof Proposition 3. (i) For a blind charge to be optimal for P, this must preferable to dropping the case, $-(1-\gamma)L - T > -\gamma H \Leftrightarrow \gamma > \frac{T+L}{H+L}$, and also preferable to own investigations, $-(1-\gamma)L - T > -c_P - \gamma T \Leftrightarrow \gamma > 1 - \frac{c_P}{T+L}$. For investigations to be a best response by J to blind charges by P, this must be more favorable than blind acquittals, $c_J - T > -\gamma H - T \Leftrightarrow \gamma > \frac{c_J}{H}$, and better than blind convictions, $-c_J - T > -(1-\gamma)\alpha H - T \Leftrightarrow \gamma < 1 - \frac{c_J}{\alpha H}$. (ii) As P charges all defendants in equilibrium, J has to form her beliefs as $\mu = \gamma$, which was considered in (i).■

Proof Proposition 4. (i) Given that P only charges the guilty defendants, for J it is always optimal to convict all charged defendants without further investigations. Given blind convictions by J, investigations are rational for P when $-c_P - \gamma T > -(1-\gamma)\alpha H - T \Leftrightarrow \gamma < 1 - \frac{c_P}{T+\alpha H}$ and $-c_P - \gamma T > -\gamma H \Leftrightarrow \gamma > \frac{c_P}{H-T}$. (ii) As P charges only the guilty defendants, $\mu = 1$ applies, as considered in (i). ■

Proof Proposition 6. (i) Given that J blindly convicts, P will respond with a blind charge if $-(1-\gamma)\alpha H - T > -c_P - \gamma T$, which gives $\gamma > 1 - \frac{c_P}{\alpha H + T}$, and $-(1-\gamma)\alpha H - T > -\gamma H \Leftrightarrow \gamma > \frac{\alpha H + T}{(1+\alpha)H}$. J´s best response to a blind charge is a blind conviction if $-(1-\gamma)\alpha H - T > -c_J - T$, which yields $\gamma > 1 - \frac{\alpha H - c_J}{\alpha H}$, and $-(1-\gamma)\alpha H - T > -\gamma H - T \Leftrightarrow \gamma > \frac{\alpha}{1+\alpha}$.■

Proof Proposition 7. (i) Given that P drops the case, J is indifferent between his strategies. For (n, dr) being the best response to J convicting blindly, first, $-\gamma H > -\gamma T - c_p$ must hold, which gives $\frac{c_P}{H - T} > \gamma$, and second, $-\gamma H > -T - (1-\gamma)\alpha H$, which yields $\frac{\alpha H + T}{(\alpha+1)H} > \gamma$. (ii) Blind convictions by J are only sequentially rational if J would prefer this outcome to blind acquittals. Thus, $-T - (1-\gamma)\alpha H > -T - \gamma H$ must also hold which yields $\gamma > \frac{\alpha}{\alpha+1}$. Also, J does not investigate the case in this subgame if $-c_J - T < -(1-\gamma)\alpha H - T \Leftrightarrow 1 - \frac{c_J}{\alpha H} < \gamma$ ■