Equal protection versus efficient security against crime:  
Differences and unintended consequences

Hans-Bernd Schäfer (Bucerius Law School, Hamburg)  
Ram Singh (Delhi School of Economics)

Abstract:

Police services are cited by economists as an example of a standard public good that should be supplied by the state to prevent market failure on account of the free-rider problem. However, from a historical perspective, the public police is a relatively new phenomenon. Even today in many countries the number of private security workers is larger than the number of public police officials. In this paper, we argue that the public police often do not aim to correct market failure, nor they tend to mimic a well-functioning market for security by reducing losses from crime to an efficient level. On the contrary, the policy goal of fighting against crimes - especially against violent ones - has moved towards ensuring equal security for all residents. This aim has gained influence from writings of constitutional law scholars, who argue that the right to equal security against crimes derives from equal constitutional rights. It has also gained acceptance as a practical police strategy.

We provide a model to examine legal and economic implications of the right to equal security against crimes. We show that ‘equal security for all’ approach toward policing is inconsistent with several other equally plausible legal objectives. It reduces efficiency as well as effectiveness of the crime fighting efforts. We show that even without discriminatory intent, the equal security-oriented policing leads to crime clearance und punishment rates that are different across police districts and also across communities. In particular, under highly plausible and realistic conditions, the equal security can result in statistical discrimination against minorities or immigrants in terms of crime punishment rates. Moreover, the adverse effects of the equal
security approach – in terms of the effectiveness of policing and discrimination - are worse than the efficient security.

**Key Words:** Public police, crime, right to equal security, non-discrimination, punishment rate, efficient policing

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**A. Loss reduction versus equal protection against crime**

Among economists, police services are a standard example of a public good, which the state must provide to prevent undersupply due to the missing demand from free riders, thus leading to a market failure. Correcting a market failure leads to public regulation and state financing and usually to mimicking a well-functioning market.¹ We show however that despite its merits - especially with regard to general crime deterrence - the public good theory of state police obscures the fact that a modern tax-financed public police provides a service which even a perfect market for police forces could not offer as its objectives differ from wealth maximization or economic efficiency. Also, the literature on the history of the police shows that public police is in historical perspective a relatively recent development and closely related to the growing scope of police tasks under the modern concept of the rule of law with a state monopoly on the use of violence. Until well into the 19th century, police in England for example was mainly privately organized and financed.² Even if the public good problem would not exist at all public police with its specific goals would be necessary because a rule of law state wants to achieve the result of equal security for all, which even a perfect market for private security cannot deliver.

Equal security for all as the mission of public police is now widely and internationally accepted not only among constitutional lawyers, but also among practitioners and in the political arena.

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² “By the eighteenth and nineteenth centuries in Britain and America, volunteer groups using private funding complemented, supplemented, or supplanted the mandatory systems of community protection. In Britain, felons associations posted rewards to apprehend criminals, assisted their members in prosecuting criminals, and sometimes hired private patrols. Victims hired private thieftakers to retrieve stolen property. In colonial America, Boston established a night watch in 1636, and watchmen became commonplace throughout the colonies. “Crime control administered by a centralized government did not exist, and responsibility for protection was thrust upon the people themselves.”” M. Rhead Enion, “Constitutional Limits on Private Policing and the State’s Allocation of Force,” *Duke Law Journal* 59 (2009): 519-53, at p. 533.
In Germany, a constitutional ‘right to security’ does not exist in the wording of the constitution, even though this has been proposed in the literature. But the constitutional court derives from several constitutional rights together with Art. 1 (1) of the constitution, which orders the state to protect human dignity, a state duty to protect basic rights such as life and health (Art. 2 (2) of the constitution) and property (Art. 14). From this doctrinal concept derives not only an objective responsibility of the state to organize protection and security against criminals but also an individual constitutional right to such protection. Because there exists also a general constitutional right to equal treatment (Art. 3 (1) of the constitution), the ‘right to security’ is – in general – a right to equal protection. It supplements the traditional concept of preserving law and order as a function of the state, which is not right based.\(^3\)

The UN security council urged (2004) “promotion of …equal security … for all inhabitants of Kosovo”.\(^4\) In the coalition agreement of the grand coalition (2018) in Germany one can read: “We do not want zones of unequal security in Germany”\(^5\) These are two examples from a long list of authoritative political statements on the policy goal of public police.\(^6\)

Public police do not mimic a perfect market for private police but reduce numbers of crimes weighted with the severity of a crime, and the severity weight is – in principle – equal for each citizen leading ideally to an equal state protection of equal fundamental rights. We call this the equal security approach of public police. This concept is a raw approximation because even in democratic rule of law states the constitutional rules relating equal fundamental rights for all citizens to police law differ across counties. It is however undisputable that the state does not follow the efficiency criterion and the related willingness to pay as a starting point for defining the mission of public police. The rationale of public police is not reducing losses from crimes based on the willingness to pay for loss avoidance. Public police has often not even the information for reducing losses, as crime statistics, on which police planning is based, contains

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\(^3\) The same fundamental difference between public and private police is mapped out by Clifford D. Shearing & Phillip C. Stenning, Private Policing (Newbury Park: SAGE, 1987); Colleran, “The Growth of Private Security and Associated Criminological Concerns,” at p. 114; Enion, “Constitutional Limits on Private Policing and the State’s Allocation of Force,” at p.550. The author argues that security and welfare aspects of policing in the USA derive from the twelfth, thirteenth and fourteenth amendment of the US constitution.


\(^6\) Harel and Parchmovoski make a similar argument and argue that fair protection against crime “imparts a duty on the state to equalize individuals’ vulnerability to crime.” This would imply more police efforts or higher punishment of crimes against those who are particularly vulnerable, like crimes related to race, gender, religion, and sexual orientation. Alon Harel & Gideon Parchmovoski, “On Hate and Equality,” Yale Law Journal 109 (1999): 507-39.
information on the numbers and not or seldom on losses from crimes, let alone on the willingness to pay for the reduction of such losses. It concentrates police forces where crime rates are relatively high. Observed public police activity is not in line with an economic theory of public police, which maintains that it corrects the public good market failure. Public police in democratic rule of law countries with good governance achieves ideally an equal state protection of those constitutional rights, for which an identical endowment of all citizens exists, like life or bodily integrity. It does not achieve an efficient reduction of the losses – including non-monetary and psychic damages – from crimes. Ideally, everybody should have the same perception of safety and security so that the fear of crimes does not have a chilling effect on his or her daily life or on the exercise of political rights like the right to assemble. This is important for individual freedom and necessary for a free and equal use of rights in a democratic society. Public police should therefore be understood as a twin sister of fundamental and equal human rights and not as a public organization for the correction of a market failure. The measure of equality for a market driven private police or a market mimicking public police is the marginal loss reduction of a dollar spent on crime prevention across all police districts and all crime categories in a state. For the public police it is an equal safety level for all residents. The state provision of security therefore differs categorically from economic regulation, which corrects market failure and tries to establish or mimic workable markets, as in antitrust law, regulation of capital markets, natural monopolies, networks or in tort law.

B. Inconsistencies of the two approaches to security

Efficiency-based police cannot achieve equal security for all residents. With regard to crimes against life, bodily integrity or the security of public places, it leads to unequal protection based on the willingness to pay in different zones with different incomes of its residents. The resulting inequality, which translates into unequal protection of equal rights becomes more severe with a more unequal distribution of wealth.

The “equal security for all” mission of the public police is not without inconsistencies either. In a market based private security, the police budget is derived automatically from the willingness to pay. On the contrary, the equal security approach does not determine the police budget. It seems to be the general opinion among constitutional lawyers, that the size of the budget is subject to a final and exclusively democratic decision in parliament if it is not
obviously much too low. This combination of two social decision mechanisms poses questions of justice and consistency too. “Equal security” with almost any police budget size cannot categorically stand alone as a definite normative concept. It can violate other important principles of justice and public policy:

1. Effectiveness. The marginal spending for police should have the same effect on crime reduction in every police district. Effectiveness guarantees that the total number of crimes in a state is minimized with a given police budget.

2. Inclusive benevolence. With public police no resident should be less secure than without public police. A stronger form of inclusive benevolence relates this policy goal to any increase of the police budget.

3. Non-discrimination. Police activities against offenders should be non-discriminatory with respect to gender, race, or religion.

4. Balance between rights. The police budget must not be increased above a level, at which an additional unit of public police spending increases security less than it would improve the realization of another human right if that unit had been spent differently.

5. Minimal efficiency. The police budget per resident in a state should not be lower than the willingness to pay in the police district with the lowest per capita income and not be higher than the willingness to pay in the district with the highest per capita income.

Public police, which try to establish equal security for all, are likely to violate one or more of these principles. An easy illustrative case is this. Assume that a municipality has two police districts, a low-income district with a high crime rate (hot spot) and a high-income district with half of this crime rate if no police exists. Assume also, for the sake of argument, that in the low-income district all offenders are black and in the high-income district they are all white. This implies that a third of all crimes are committed by whites. Then equal security in both districts requires the concentration of police in the hot-spot area until crime rates are the same in both districts. This violates (3) because all arrested and convicted criminals are blacks. It also violates (2) because concentrating the police force in the hot spot leads to crime diversion, which makes residents in the rich district less safe than they would be without the existence of public police. If one wants to avoid this by evenly spending the budget in both districts, this
violates (1) and the principle of equal security for all residents itself. If the police budget is increased to such a high level that the police can substantially reduce crimes in both districts and racial discrimination does not happen, this might not violate (1), (2) or (3) but it might violate (4).

Achieving equal security for all residents, including those targeted by hate crimes, might require concentrating public security spending on the targeted individuals or members of a targeted religious community. Assume that the victims of hate crimes are rich and live in the district with the highest incomes. Then achieving equal security across all police districts might require a level of police spending in favor of the potential victims of the hate crimes, which is higher than their own willingness to pay for police. This would violate minimal efficiency (5).

Equal security in all police districts can violate the principle of effectiveness (1). The latter guarantees that public police minimize the number of crimes in the state, given the police budget. Depending on the relation between police spending in different police districts and crime rates, this can lead to zones of unequal security in a state.

It is therefore inevitable that the goal of equal security for all – as the concept of efficient security – cannot be categorical but must be traded off against other principles and reasonable policy goals.

C. Efficient security versus equal security, market versus state.

Economic efficiency would require to concentrate police forces in such a way that the sum total of all losses (including the money value of pain and suffering) from crime and costs of crime prevention are minimized. A perfect private market for police forces could achieve this. A perfectly working state police could also achieve this.

The protection of basic rights requires equal protection of all against crimes which violate equal basic rights. A perfectly working state police can achieve this. A perfect private market and privately financed market for security cannot achieve this.

Economic efficiency and equal protection against crime are the end points on a scale. Here are some crime categories, which either should fought under the efficiency rationale or under the equal protection rationale or under a mixture of both.
<table>
<thead>
<tr>
<th>Type of crime</th>
<th>Type of crime prevention (cost benefit or/and equal rights protection)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tax evasion</td>
<td>Cost benefit, because problems of human rights do not arise, except indirectly because state functions are negatively affected. Therefore, the cost benefit approach is appropriate even from a human rights point of view.</td>
</tr>
<tr>
<td>Insurance Fraud</td>
<td>Cost benefit, again human rights are only indirectly affected</td>
</tr>
<tr>
<td>Shop lifting, theft</td>
<td>?</td>
</tr>
<tr>
<td>All violent crimes, murder, armed robbery, robbery, rape, blackmail, extortion, kidnapping</td>
<td>Equal protection of all citizens</td>
</tr>
<tr>
<td>High treason, state treason</td>
<td>Cost benefit, because the state is the only direct victim and has to minimize the overall effects of such crimes.</td>
</tr>
<tr>
<td>Crimes against competition laws, for instance cartel formation</td>
<td>Cost benefit. The social losses from such crimes should be minimized in terms of efficiency losses. It might be better to concentrate police forces on the detection of one large cartel than on the detection of three small cartels. Also, the rationale of competition laws is to restore economic efficiency as much as possible.</td>
</tr>
</tbody>
</table>

D. Crime reduction with public police or private civilian security
Security provided by the police depends on three factors, first the policy target, which can be optimal loss from crime or equal security (or a combination of both), second the division of police expenditures into repression and prevention and the overall police expenditures. The term “repression” encompasses all activities, which lead to the detection and punishment of offenders, including fact finding, interrogation, using violence against offenders, arresting and jailing, administrative fines, and preparing criminal procedures leading to a general deterrence effect and in the case of prison sentence to incapacitation. Prevention covers all activities which reduce crime by observing and patrolling and therefore by decreasing the probability of a successful crime.

We analyze optimal police expenditure first for one police district and then extend to more than one district including interdependencies between them.

E. Optimal crime prevention and crime deterrence
Let $L$ denote total losses from crime in the police district under consideration and $N$ the number of offences. We assume for convenience that one criminal commits one crime, Therefore $N$ is also the number of criminals. $v$ is the damage per crime which is the sum total of a pecuniary damage as well as physical and psychic pain from crimes and their chilling effects. We assume that these moral damages can be reliably expressed in money terms and therefore represented on the same dimension as pecuniary losses. Damages per victim $v$ are assumed to be the same, which presupposes at this stage of analysis that victims are homogeneous and have the same income. Police expenditures are $x$. Total losses from crime are then

\[ L = N(x)v \] with $L'(x) < 0$ and $L''(x) < 0$, denoting decreasing marginal returns from more police expenditures. Total costs from crime are then

\[ L = N(x)v + x \]
A well-functioning private police would equalize marginal total losses and marginal police expenditures as denoted by the first order condition of the total cost function.

\[ vN'(x) = 1. \]

Right based public police do not reduce losses from crimes to an efficient level but try to achieve an equal probability for all residents of becoming victim of a crime. It would reduce the number of crimes weighted with their severeness, for instance with a monetary equivalent of average financial and non-financial losses from a crime or with a utilitarian weighing. For a police, which reduces “numbers of crimes” \( N \) is then the number of a representative crime with a weight of one. But this number would include all other crimes to with a weight of less or more than 1. If armed robbery were chosen as the representative crime one more such crime increases the crime number \( N \) by one and for more or for less severe crimes by more or less than 1.

Without police there exists a maximal number of crimes (and offenders) \( m \) which is only constrained by private and personal vigilance and physical investment against crimes, which are left out of this analysis. Therefore \( m \) is regarded as a parameter. We assume that increasing police expenditures \( x \) makes crimes more expensive, which continuously reduces crimes. There exists no threshold value of \( x \), at which crimes disappear. Unlike in the classical Becker model we assume that individuals are not homogeneous but have different costs and from committing the same crime. Therefore at any level of police expenditures there exists a positive number of crimes. Marginally higher police expenditures always lead to abstention from crime for some individuals but never for all.

**F. Optimal police expenditure for a preventive and repressive police**

Police reduces crimes with preventive and repressive methods. Crime prevention consists of all ex ante activities like patrolling, observation, and protecting persons and objects. Prevention reduces crimes but usually does not lead to arrest, conviction, and punishment and therefore cannot generate general deterrence. Crime repression consists of detective and investigative tasks ex post, after a crime occurred, to punishment and therefore to
general deterrence, which again reduces the number of crimes. Total police expenditures are therefore split into preventive ex ante and investigative ex post police duties. If \( q \) is the share of total expenditures for prevention and \( 1 - q \) the share for detection and repression total police expenditures are

\[
x = qx + (1 - q)x.
\]

Repression causes the quota of detected and punished crimes, that is the probability of being punished for a committed crime, which would be zero with purely preventive police. We assume therefore that with other things equal an increasing quota of police expenditures for repression increases the effect of general deterrence and decreases the effect of prevention.

\[
L = Nv = \frac{mv}{(qx)^a \cdot [(1 - q)x]^{(1-a)}} + x
\]

\( q \): quota of police expenditures allocated for crime prevention.
\( 1-q \): quota of police expenditures for crime repression.

The denominator is a Cobb-Douglas function with constant returns to scale. Remember that for mathematical reasons the denominator in formula (1) cannot be smaller than 1. Otherwise \( m \) (the maximal number of crimes, when police does not exist) is not defined.

The optimal values for \( q = q^* = a \) and \( x = x^* \) can be determined with analysis. Replacing \( q \) with \( q^* = a \) in formula (1) and differentiating with respect to \( x \) yields

\[
L'(x) = \frac{mv[(1-a)x]^a}{(1-a)x^2(ax)^a} - 1 = 0
\]

This yields the optimal police expenditures \( (x^*) \)

\[
x^* = \frac{\sqrt{mv(1 - q)^{a-2}}}{a^a}
\]
The optimal split of police expenditures between prevention and repression is exclusively determined and equal to $a$.

$$q^* = a. \tag{2}$$

$a$ is a parameter of relative effectiveness of repression in comparison to prevention. No other parameter or variable in the model can change the value of $q^*$ which is therefore a parameter itself, provided the police works effectively and the technology of police activity is held unchanged, which we assume throughout the paper. The production function for police work changes therefore from

$$(qx)^a((1 - q)x)^{1-a} \text{ to } (ax)^a((1 - a)x)^{1-a}$$

This implies that

$$(ax)^a((1 - a)x)^{1-a} = a^a(1 - a)^{1-a}x.$$  

This function is only defined if $a \in (0,1)$. It is a linear function of $x$ with a fixed coefficient. We can therefore replace this formula with a variable, which multiplies the police expenditures with a constant factor. If we normalize this factor to 1 this reduces the complexity of the model but not the analysis because this is not different from expressing the police expenditure in another currency. The $x$ in the subsequent analysis contains however the information that police expenditures are optimally split between repression and prevention of crime.

G. Crime fighting for Efficient Security

I. Districtwide police expenditure and crime rates with efficient police expenditures

So far, the analysis could not arrive at a difference between an efficiency based and an “equal security for all” based police as it does not include the possibility of different zones and police districts with potentially different levels of crime and unequal security. In the
next section we move the analysis to a state in which more than one police district exists in which crime rates can differ across police districts. Two different police districts are introduced which can differ regarding the wealth of their residents and crime levels. Police forces are allocated and organized districtwide. If citizens in 2 different districts have different incomes this implies higher losses per crime in the high-income district compared with the low-income district. Let \( i \in \{1, 2\} \) denote a high-income and a low-income district then \( v_1 > v_2 \) holds for 2 reasons. Monetary losses per property crime are higher. Non-monetary losses are also higher, not in terms of their psychic severity but in monetary equivalents. If the maximal number of crimes (without police) is \( m_1 \) and \( m_2 \) for district 1 and district 2 the rich district attracts more crimes without police \( (m_1) \) than the poor district \( (n_2) \). This implies \( n_1 > n_2 \). A perfect market for police would minimize Total losses from crime in both districts. In formula (3) we skip the production function for the police for the reasons explained above.

\[
L = L_1 + L_2 = \frac{m_1 v_1}{x_1} + x_1 + \frac{m_2 v_2}{x_2} + x_2
\]

The first order conditions for \( x_1 \) and \( x_2 \) yield

\[
x_1^* = \sqrt{m_1 v_1}, \quad x_2^* = \sqrt{m_2 v_1},
\]

Imputing these values into formula 3 allows to calculate the efficient losses from crime and the efficient crime numbers. For \( i \in \{1, 2\} \) we get

\[
L_1^* = \frac{m_1 v_1}{\sqrt{m_1 v_1}}; \quad N_i^* = \frac{L_2^*}{v_2} = \frac{m_2}{\sqrt{m_2 v_2}}
\]

In a perfect private security market this solution is also the unique market equilibrium. A state police would achieve the same outcome, if it regarded police as a public good, which it provides for the only reason of a market failure and tries to mimic an efficient market for private security. Provided that the 2 districts have the same number of residents, which we assume throughout the paper the efficient number of crimes will be different across districts, if \( m_1 \neq m_2 \) and/or \( v_1 \neq v_2 \).
A market oriented or market mimicking police achieves efficient losses from crime, but not equal security or (in the model) equal numbers of crime in all districts. Equal security is only achieved accidentally when $n_1v_1 = n_2v_2$ applies.

II. Discriminating effects of efficient security

In the next step we change assumptions to analyze unintended consequences of police practice on minorities with a potentially relatively high crime rate. It is not straightforward that tensions might exist between efficient security and non-discrimination. In this section we show otherwise. For analytical clarity we rule out any racist or sexist or similar cause for intentional discrimination and -to paraphrase Gary Becker- we assume that nobody in the police force has “a taste for discrimination”. This excludes targeting members of minority groups for such reasons. We assume that all police forces adhere loyaly to the policy of efficient security and show that even under this ideal and unrealistic assumption discrimination can still arise. If a minority group has a higher crime rate than the majority police concentration in police districts with minority population can lead to a clime clearance and punishment quota of offenders from the minority, which is higher than the punishment quota of offenders within the majority population.

Our concept of discrimination is therefore that the detection and punishment quota of minority offenders is higher than the detection and punishment quota of offenders from the majority offenders.

This concept of discrimination, which we use throughout the paper, does not define discrimination in a legal sense, because it describes a discriminating effect without discriminating intent. The legal status of this form of discrimination differs across countries. We use it anyway and leave it open when and under what conditions this result might be of legal relevance or whether it is only an unwanted adverse effect without a legal consequence. The above concept of discrimination is however in line with the EU antidiscrimination directive, which differentiates between direct discrimination, based on discriminatory intent and indirect discrimination. “(b) Indirect discrimination shall be taken to occur where an apparently neutral provision, criterion or practice would put
persons of a racial or ethnic origin at a particular disadvantage compared with other persons, unless that provision, criterion or practice is objectively justified by a legitimate aim and the means of achieving that aim are appropriate and necessary.”

This definition of the Council Directive acknowledges the possibility that a legitimate state aim can have indirect discriminatory consequences, which are not illegal but constrained by the proportionality principle of public and constitutional law. We now assume that 2 types of residents live in a state, members of the majority and minority population. Minority members are immigrants and have a higher crime rate only because they are in the average much younger than the members of the majority group. Crime statistics tell that crimes concentrate among younger people between 12 and 34.  

We assume that this is the only element of divergence between the 2 groups. We assume again 2 police districts. In district 1 all residents are from the majority population. District 2 has again the same number of residents but a quota of \( s \in (0,1) \) is from the minority. Consequently, the share of the majority population of the total population in district 2 is \((1-s)\). The number of crimes is again equal to the number of criminals. The number of crimes without the existence of police is then \( m_1 \) in district 1. District 2 is a mixed area with the same number of inhabitants. But now a quota of \( s \) with \( s \in (0,1) \) denotes the quota of the minority population and has therefore a maximal crime number of \( sm_2 \) with \( m_2 > m_1 \). A quota of \((1-s)\) in district 2 denotes the quota of the majority population in district 2, which has the same tendency to criminal behavior as the majority population in district 1. The maximal number of crimes of the majority population in district 2 without police is therefore \((1-s)m_1\). We also assume that the losses from a crime are the same across districts, \((v = v_1 = v_2)\), which focuses the analysis on the different tendency to commit crimes by the majority and minority population. The losses from crimes and the costs of crime fighting are then

\[
L = L_1 + L_2 = \frac{m_1v}{x_1} + x_1 + \frac{(1-s)m_1v + sm_2v}{x_2} + x_2
\]


\( ^8 \) Statista (2021), Prevalence rate of violent crime, by age U.S. 2019 | Statista
Differentiating $L$ with respect to $x_1$ and $x_2$ yields the first order conditions for optimal police expenditures, which yields

$$x_1^* = \sqrt{m_1 v} \text{ and } x_2^* = \sqrt{m_1 v(1-s)} + m_2 vs,$$

From this it is easy to see the condition under which efficient police expenditures in district 2 are higher than in district 1.

$$x_2^* > x_1^* \text{ if } m_2 > m_1$$

This applies by assumption that maximal crime numbers are higher in the minority population than in the majority population. The efficient losses from crime are now

$$L^* = L_1^* + L_2^* = \frac{m_1 v}{\sqrt{m_1 v}} + \frac{(1-s)m_1 v + sm_2 v}{\sqrt{m_1 v(1-s) + m_2 vs}}$$

The efficient number of criminals (criminal offenses) is then

$$N^* = N_1^* + N_2^* = \frac{L_1^*}{v} + \frac{L_2^*}{v} = \frac{m_1}{\sqrt{m_1 v}} + \frac{(1-s)m_1 + sm_2}{\sqrt{m_1 v(1-s) + m_2 vs}}$$

In police district 2 the efficient number of criminals among the majority population is

$$N_{2,1}^* = \frac{(1-s)m_1}{\sqrt{m_1 v(1-s) + m_2 vs}}$$

The efficient number of criminals among the minority population is

$$N_{2,2}^* = N_{\text{minority}}^* = \frac{sm_2}{\sqrt{m_1 v(1-s) + m_2 vs}}$$
The efficient number of criminals in the majority population is

\[ N_1^* + N_2^* = N_{majority}^* = \frac{m_1}{\sqrt{m_1 v}} + \frac{(1 - s)m_1}{\sqrt{m_1 v(1 - s) + m_2 v s}} \]

Police forces are higher in district 2 than in district 1 because the immigrant minority is younger and therefore commits under equal crime fighting intensity more crimes than the majority population. Unequal distribution of police forces as such is not discriminatory in the sense described above but related to differential crime clearance and punishment rates.

We now introduce the crime clearance rate. We assume for convenience that this rate is equal to the punishment rate. Without police this rate is 0. It increases with more police expenditures and can at most reach a value of 1. We get therefore for the clearance rate \( 1 > r \geq 0 \), \( r \) increases with police expenditure and approaches 1 if police expenditures increase beyond all limits. The following equations include these properties. \( c \) is a constant parameter. For police district 1 and 2 the efficient crime clearance rates are then:

\[ r_1^* = \frac{x_1^*}{x_1^* + c} = \frac{\sqrt{m_1 v}}{\sqrt{m_1 v + c}} \quad \text{and} \]
\[ r_2^* = \frac{x_2^*}{x_2^* + c} = \frac{\sqrt{m_1 v(1 - s) + m_2 v s}}{\sqrt{m_1 v(1 - s) + m_2 v s + c}} \]

One can see from a comparison of (8) and (9) that \( r_2^* > r_1^* \) because \( m_2 > m_1 \) by the assumption that the maximal crime rate among the minority population of immigrants is higher. As private security or market mimicking police minimizes the sum of losses from crime and costs of crime fighting, we can now calculate the clearance rates for the majority and the minority population.
The punishment rate of the offenders from the majority ($r_{\text{majority}}$) is the total number of punished offences (and offenders) in the majority population ($N_{2,1}$) divided by all offences committed by members of the majority group.

\[
r_{\text{majority}} = \frac{r_1^* N_1^* + r_2^* N_{2,1}^*}{N_1^* + N_{2,1}^*}
\]

The equivalent punishment rate from the minority population is

\[
r_{\text{minority}} = \frac{r_2^* N_{2,1}^*}{N_{2,1}^*} = r_2^*
\]

Therefore, a discriminatory effect of the efficient expenditures for police against the minority population emerges if

\[
\frac{r_1^* N_1^* + r_2^* N_{2,1}^*}{N_1^* + N_{2,1}^*} < r_2^*
\]

From this one can see that the condition for discrimination under efficient security is met if

(10) \hspace{1cm} r_1^* < r_2^*

This is the case (see formulas (8) and (9)). Therefore, a well-functioning private security market or a market mimicking public police leads to a discriminatory or adverse effect in the sense described above. With private security or a market mimicking police the crime clearance and punishment rate of offenders within the minority population becomes higher than for the offenders within the majority population.

H. Crime fighting for equal security
I. Districtwide police expenditures and crime rates with equal security

In a private security market, decisions on the police expenditures of $x_1$ and $x_2$ are taken districtwide that is independent from district to district. A state, which wants to achieve equal security must take two decisions, fix a police budget and distribute the police expenditures to the different districts. Unlike under efficient security the later decision is interrelated and simultaneous.

The decision on a police budget is a political parliamentary decision. Unlike for an efficient police welfare economics cannot inform politics about the best size of the police budget. The constitutionally informed policy target of equal security policy does not much constrain the size of $b$. For the police administration unlike for private security the budget is therefore basically a constant and unchangeable political parameter. On the contrary, a market mimicking state would fix a budget of the size $b = \sum_i x^*_i$.

The total police budget $(b)$ is based on a democratic parliamentary decision. The police department then decides how to split the budget across the 2 police districts. We denote the share of the budget going to police district 2 as $p$. Consequently we get

$$b = (1 - p)b + pb$$

(11)

Equal security in both district requires

$$N_1 = N_2 = \frac{L_1}{v} = \frac{L_2}{v}$$

(12)

Efficient police, which minimizes the sum of losses from crime and costs of police, will move optimal police expenditures of $x^*_1$ to district 1 and $x^*_2$ to district 2. Both values are calculated with analysis by two independent steps. There exist no interdependencies between police districts. A police force, which wants to equalize crime rates, that is numbers of crimes in the two districts with assumedly equal population size must fix police expenditures in line
with formula (12). The police authority must distribute the police budget among the two districts in such a way that the number of crimes becomes equal in both districts.

From this it follows immediately that unlike for the efficient police the quota of the budget, which goes to district 1 \((1-p)\) and the quota which goes to district 2 \((p)\) and consequently police expenditures for both district must be simultaneously fixed. The number of crimes in both districts must become equal.

\[
N^e_1 = N^e_2
\]

The equalizing number of crimes is then \(N^e\).

\[
(13) \quad N^e = 2N^e_1 = 2N^e_2 = N^e_1 + N^e_2 = \frac{m_1}{(1-p)b} + \frac{m_1(1-s) + m_2s}{p b}
\]

The value of \(p\), that is the budget share in district 2, which equalizes the number of crimes in both districts \((p^e)\) follows directly from solving (13) for \(p\).

We now compare the number of crimes under equal security and under efficient security. For this it is useful to first compare the budget shares \(p^e\) and \(p^*\) under the two different policies

\[
(14) \quad p^e = \frac{(m_2-m_1)s+m_1}{(m_2-m_1)s+2m_1'},
\]

with \(p^e'(s)>0\), \(p^e''(s)<0\), \(p^e(0)=0.5\) and \(p^e(1)=\frac{m_2}{m_1+m_2}\)

Note that the budget share for district 2, which equalizes crime numbers in both districts \((p^e)\) is not dependent on the budget but only on the minority quota in district 2 and the sizes of maximal crime numbers (without police). It is 0.5 if \(s=0\) and rises to a maximal value when \(s\) increases to 1.
The comparable share of expenditures under efficient crime reduction \( (p^*) \) is for the police district 2

\[
p^* = \frac{\sqrt{m_2v - m_1v + m_1v}}{\sqrt{m_1v} + \sqrt{m_2v - m_1v + m_1v}} \text{ with values for } s \text{ of } 0 \text{ and } 1
\]

we get

\[
p^*(0) = 0.5; \quad p^*(1) = \frac{\sqrt{m_2v}}{\sqrt{m_1v} + \sqrt{m_2v}}; \quad p^*'(s) > 0; \quad p^*''(s) < 0.
\]

\( p^* = p^e = 0.5 \) if no minority population exists in district 2. This is obvious, because both districts are assumed to be homogeneous except if a minority population lives in district 2.

For comparing the two alternative police regimes the following result is important.

\[
p^e > p^* \text{ if } s > 0 \text{ and if } m_2 > m_1 \text{ (proof, see appendix)}
\]

The equivalent for the budget share for police district 2 \( (p^e) \) of the equal security police is

\[
p^s = \frac{x_2^*}{x_1^* + x_2^*} = \frac{\sqrt{m_2 - m_1} + m}{\sqrt{m_1} + \sqrt{m_2 - m_1} + m_1}
\]

For reasons of comparability of equal versus efficient security, we assume that the police budget under equal security is equal to the efficient police expenditures under private security or market mimicking public police. \( b = b^* = x_1^* + x_2^* \).

\[
N_2^e < N_2^* \text{ or } \frac{m_1(1-s)+m_2s}{p^eb^*} < \frac{m_1(1-s)+m_2s}{p^*b^*} \text{ because } p^e > p^*
\]

The number of crimes in the high crime district is -with an equal security police-lower than with a market mimicking efficient police. By the same token we get the crime rate for the low crime district 1.
\[(19) \quad N^e_1 > N^*_1 \text{ or } \frac{m_1}{(1-p^e)b^*} < \frac{m_1}{(1-p^*)b^*}, \text{ because } (1 - p^e) > (1 - p^*) \]

In comparison equal security leads to a higher crime rate in the low crime district and a lower crime rate in the high crime district compared with the police expenditures under an efficiency rationale.

From (13)-(18) it can also be deduced that

\[(20) \quad N^e > N^* \]

The total number of crimes with an equal security police is higher than with a market mimicking police. This loss of overall effectiveness of crime fighting is a cost of reaching the policy target of zones of equal security in the state. (Proof, see Appendix). This follows from the assumption that the maximal crime rate among the minority population is higher than among the majority population (see appendix).

A further observation is that in this model the difference between \(p^e\) and \(p^*\) increases with the ratio \(m_2/m_1\) and is -other things equal- the same for a particular ratio, independent from the absolute values of \(m_2\) and \(m_1\). The difference also increases with \((s)\), the quota of minority population in sector2. These two factors of an equal security policy move police efforts more into the zones of relatively high crime as compared with an efficient security, which tries to minimize losses from crime.

We have assumed that the losses from one crime are the same across the 2 district. It is obvious that the described effect increases, if the losses from crime are lower in sector 2 than in sector 1 as this would leave \(p^e\) unchanged but decrease \(x^*_2\) and therefore decrease also the efficient \(p^*\). This would cause a further concentration of police efforts in district 2 under the equal security rationale as compared with efficient security.

II. Discrimination effects of equal security
To show whether the distribution of police expenditures among police districts can cause discrimination we exclude again direct forms of police discrimination against minorities. According to our definition discrimination happens if the crime clearance rate among the crimes committed by the minority population is higher than the equivalent clearance rate among crimes committed by the majority population. We ask whether equal security can lead to discrimination in this sense.

Like in the last section we assume the following formula for the crime clearance rate in both sectors. The crime clearance rate is however not the efficient but the equal security rate \( r^e \).

\[
(17) \quad r^e_1 = \frac{x_1^e}{x_1^e + c} = \frac{(1-p)b}{(1-p)b+c} \quad \text{and} \quad r^e_2 = \frac{x_2^e}{x_2^e + c} = \frac{p^e b}{p^e b+c}
\]

As the police equalizes the number of crimes in both districts, we can now write the punishment rates for districts 1 and 2. For the majority population the punishment rate, which equalizes security over different zones is the number of punished crimes in both sectors divided by all crimes committed by offenders from the majority population. That is the equalizing punishment rate of the majority \( r^e_{\text{majority}} \) is the total number of punished offences by the majority divided by all offences committed by the majority.

Note that

\[
r^e_{\text{majority}} = \frac{N^e_1 r^e_1 + N^e_2 (1-s) r^e_2}{N^e_1 + N^e_2 (1-s)} = \frac{r^e_1 + (1-s) r^e_2}{1+(1-s)} \quad \text{as} \quad N^e_1 = N^e_2.
\]

The punishment rate of the immigrants is the detected and punished number of all crimes committed by offenders from the minority divided by the number of all crimes committed by offenders from the minority.

\[
r^e_{\text{minority}} = \frac{N^e_{\text{str}}}{N^e_{\text{str}}_2} = r^e_2
\]
We defined discrimination as a higher punishment rate for the offenders from the minority compared with the offenders from the majority.

\[
\frac{r_1 + (1-s)r_2}{1 + (1-s)} < r_2 \text{ or } r_1 + r_2 - r_2 s - < 2r_2 - r_2 s
\]

It follows immediately that discrimination occurs as result of a human right based policy of equal security if \(r_2 > r_1\).

\[
(17) \quad r_{\text{minority}} > r_{\text{minority}} \text{ if } r_2 > r_1.
\]

This condition is met if the maximal crime number without police is higher among the minority population compared with the majority population. In the model discrimination is therefore a consequence of an equal security target for public police.

We conjecture that the condition for discrimination of the minority population would also apply in a more general model with \(n\) districts and with minorities living in all \(n\) districts as long as the minority population is not equally spread among all police districts.

Discrimination under the equal security policy increases with higher crime rates among minorities and higher geographical concentration and a higher quota of minorities in a particular region.

I. The discriminatory effect of equal versus efficient security in comparison

In this section we show that discriminatory effects of equal security are more severe than those of efficient security. We have already shown that discrimination occurs with both policy goals under the same conditions, namely that there exists a minority population with a higher maximal crime rate than that of the majority population. In this section we show that the comparative discriminatory effect under equal security is higher in our model than under efficient security.

If for the purpose of comparability we again assume that total police expenditures are the same under both policies
\[ b = b^* = x_1^* + x_2^* \]

The efficient police expenditure quota for district 2 is \[ p^* = x_2^* \frac{x_2^*}{x_1^* + x_2^*} < p^e \] This implies that with the same overall police expenditures the expenditures in district 2 are higher under the equal security policy than under efficiency oriented policy. Therefore, the crime clearance rate in district 2 must also be higher. \[ r_2^e > r_2^* \] As the crime clearance rate in district 2 is equal to the minority crime clearance rate the result is \[ r_{minority}^e > r_{minority}^* \]

In the high crime district 2 the crime clearance and punishment rate is higher under the equal security policy than under the efficient security policy.

In district 2 the crime clearance rate for the minority is relatively high under equal security policy. But the same applies for the majority citizens, who live in district 2. This is not discriminatory according to our definition of discrimination. But in district 1, in which only members of the minority population live, the police force must be lower under equal security as compared with efficient security. This follows again from the relative shares of total police expenditures, which go to police district 1. These shares are

\[ 1 - p^* = \frac{x_1^*}{x_1^* + x_2^*} < 1 - p^e \frac{x_1^e}{x_1^e + x_2^e} \]

This condition guarantees that police expenditures are under equal security lower in district 1 than under efficient security and consequently crime rates are higher and crime clearance rates are lower than under efficient security. This implies that the crime clearance rate in district 1 is lower under equal security than under efficient security. It also implies that the crime clearance rate among the offenders from the minority is higher with an equal security compared with efficient security.

The total comparative crime clearance rates of the majority under equal and efficient crime fighting is
\[ r_{\text{majority}}^* = \frac{r_1^* N_1^* + r_2^* N_1^*(1 - s)}{N_1^* + N_1^*(1 - s)} > r_{\text{majority}}^e = \frac{r_1^e + r_2^e (1 - s)}{1 + (1 - s)} \]

A switch from efficient to equal security policy has two effects. First, it increases the crime clearance rate among those members of the majority population, who live in district 2. This implies that even though the crime clearance rate increases for the offenders among the minority a switch from efficient to equal security has not a discriminatory effect in comparison to that part of the majority population, which lives in district 2. Second the crime clearance rate in district 1 \((r_1^e)\) decreases. As by assumption all inhabitants of district 1 belong to the majority population the overall effect of a switch from efficient to equal security is therefore discriminatory. This implies in the model that equal security comes at the cost of more discrimination, which is not the result of discriminatory intent but a side effect of a policy, which is itself based on human rights.

The intuition behind this result is this. Assume that in an initial situation all residents in a state are homogeneous. This implies that all police districts, which have by assumption the same number of residents, are endowed with an equal share of all expenditures for the police, no matter whether the policy target is efficient or equal security. Now assume that the initial state changes and that some districts have immigrant population, whose crime rate is higher because immigrants are in the average younger. The reaction of efficient security is then, to increase the police expenditures in the now mixed districts until marginal losses from crime and marginal police costs of crime are again minimized in every district. Therefore, police expenditures in the districts without immigrants remain unchanged. The equal security policy shifts however resources from the low crime to the high crime sectors, which increases the crime clearance rate there and decreases clearance rates in the police districts without immigrants. By the same token crime numbers go down in the high crime districts and increase in the low crime districts. This effect would also occur if for the reason of comparability, one assumes that the overall budget of the equal security police increased by the same amount as total expenditures of an efficiency oriented police would do. If the equal security police would spend only the budget increase, which occurs under efficient security as a consequence of the existence of a minority population in the high crime sectors this would not lead to equal security. An additional shift of police resources from the low to the high crime sector is necessary to achieve equal security by increasing the crime
rates in the low crime districts and reducing them in the high crime districts until both are equal. This leads to an additional discriminating effect against offenders from the minority of an equal security aim as compared to efficient security.

J. A numerical example

This section illustrates the model and compares with a numerical example different outcomes under either equal or efficient security. It makes the tradeoffs visible, which might occur under the two policy aims in terms of crime losses, crime numbers, crime clearance rates and discriminatory effects between a majority and minority population of offenders. To make comparison between the two police strategies possible we assume again that the budget of a public police, which aims for equal security for all is equal to the efficient police expenditures under a rationale of efficient security.

The population share of the high crime minority in sector two is s=0.5
The number of crimes committed be the minority without the existence of the police is \( m_2s = 3000 \times 0.5 \). The number of crimes committed by the majority population without police in district 1 is \( m_1 = 1000 \). The total number of crimes committed by the majority population is \( m_1 + m_1s = 1000 + 0.5 \times 1000 = 1500 \). The loss from a crime is 5. The parameter for calculating the crime clearance rate is \( c=50 \).

<p>| Table 1. A Numerical Example, Comparison of efficient and equal security against crime |
|-----------------------------------------------|---------------|----------------|
| Total police expenditures ( (x_1^* + x_2^* = b) ) | 171 | 171 |
| Police expenditures in district 1 | 71 | 57 |
| Police expenditures in district 2 | 100 | 114 |
| Losses from crime in police district 1 | 71 | 87.5 |
| Losses from crime in police district 2 | 100 | 87.5 |
| Total losses from crime | 171 | 175.5 |
| Number of crimes in police district 1 | 14.2 | 17.5 |</p>
<table>
<thead>
<tr>
<th>Number of crimes in police district 2</th>
<th>20</th>
<th>17.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of crimes</td>
<td>34.2</td>
<td>35.1</td>
</tr>
<tr>
<td>Crime clearance and punishment rate in police district 1 in percent</td>
<td>58.7</td>
<td>53.3</td>
</tr>
<tr>
<td>Crime clearance and punishment rate in police district 2 in percent</td>
<td>66.7</td>
<td>69.5</td>
</tr>
<tr>
<td>Average crime clearance and punishment rate</td>
<td>62.7</td>
<td>61.4</td>
</tr>
<tr>
<td>Crime clearance and punishment rate of offenders from the majority population in percent</td>
<td>61.4</td>
<td>58.7</td>
</tr>
<tr>
<td>Crime clearance and punishment rate of offenders from the minority population in percent</td>
<td>66.7</td>
<td>69.5</td>
</tr>
</tbody>
</table>

K. Appendix

I. Proof that $p^e > p^* \text{ if } s > 0 \text{ and if } m_2 > m_1$

Proposition:

$$\frac{(m_2-m_1)s + m_1}{(m_2-m_1)s + 2m_1} > \frac{\sqrt{(m_2 - m_1)v s + m_1v}}{\sqrt{m_1} + \sqrt{(m_2 - m_1)s + m_1}} \text{ if } m_2 > m_1 \text{ and } s > 0$$

$$\frac{\sqrt{m_1} + \sqrt{(m_2 s - m_1 s + m_1)}}{\sqrt{(m_2 - m_1)s + m_1}} > \frac{(m_2-m_1)s + 2m_1}{(m_2-m_1)s + m_1}$$

$$\sqrt{m_1} + \sqrt{(m_2 s - m_1 s + m_1)} > \frac{(m_2-m_1)s + 2m_1}{(m_2-m_1)s + m_1} \sqrt{(m_2 - m_1)s + m_1}$$
\[(\sqrt{m_1} + \sqrt{(m_2 - m_1)s + m_1}) \sqrt{(m_2 - m_1)s + m_1} > (m_2 - m_1)s + 2m_1\]

\[(\sqrt{m_1} \ast \sqrt{(m_2 - m_1)s + m_1}) + (m_2 - m_1)s + m_1 > (m_2 - m_1)s + 2m_1\]

\[\sqrt{(m_2 - m_1)s + m_1} > \sqrt{m_1}\]

\[(m_2 - m_1)s + m_1 > m_1\]

\[(m_2 - m_1)s > 0 \text{ if } s > 0\]

\[m_2 > m_1.\]

II. Equal security and reduced effectiveness of police work

Police can aim at minimizing the total number of crimes in a state.

\[N = \frac{L}{v} = \frac{N_{1,\text{max}}}{(1 - p)b} + \frac{N_{1,\text{max}} (1 - s) + N_{2,\text{max}} s}{p b}\]

\[p^N = \frac{((N_{1,\text{max}}^2 + N_{2,\text{max}}N_{1,\text{max}}) s^2 + N_{2,\text{max}}^2)^{0.5} + (N_{1,\text{max}} + N_{2,\text{max}}) s + N_{1,\text{max}}}{(N_{2,\text{max}} + N_{1,\text{max}}) s}\]

Differentiating \(N\) with respect to \(p\), the first order condition yields budget shares for police expenditures in districts 1 and 2 which minimize the total number of crimes \((p^N)\) with a given budget.